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Oxford Cambridge and RSA

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics
Sample Question Paper

Model Solutions

Date – Morning/Afternoon

Version 2

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

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INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r|<1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1\, a^{n-1}b + {}^nC_2\, a^{n-2}b^2 + \ \dots\ + {}^nC_r\, a^{n-r}b^r + \ \dots\ + b^n \qquad (n \in \mathbb{N}),$$

where $^nC_r = {}_nC_r = \begin{pmatrix} n \\ r \end{pmatrix} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ \dots\ + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \ \dots \qquad \left(|x|<1,\ \ n \in \mathbb{R}\right)$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \dfrac{u}{v}$, $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Differentiation from first principles

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)\left(f(x)\right)^n dx = \frac{1}{n+1}\left(f(x)\right)^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

Small angle approximations

$\sin\theta \approx \theta, \cos\theta \approx 1-\tfrac{1}{2}\theta^2, \tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \frac{1}{2})\pi)$

Numerical methods

Trapezium rule: $\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Standard deviation

$$\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} \quad \textbf{or} \quad \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n,\ p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $P(Z \le z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Section A: Pure Mathematics

Answer **all** the questions

1 **(a)** If $|x| = 3$, find the possible values of $|2x-1|$. **[3]**

1a) If $|x| = 3$, $x = 3$ or $x = -3$

If $x = 3$, $|2x-1| = |6-1| = \underline{5}$

If $x = -3$, $|2x-1| = |-6-1| = \underline{7}$

(b) Find the set of values of x for which $|2x-1| > x+1$.
Give your answer in set notation. **[4]**

b) Either $2x - 1 > x + 1$

$x > 2$

or $2x - 1 < -x - 1$

$3x < 0$

$x < 0$

$\{x : x < 0\} \cup \{x : x > 2\}$

2 **(a)** Use the trapezium rule, with four strips each of width 0.25, to find an approximate value for $\int_0^1 \frac{1}{\sqrt{1+x^2}}\,dx$. **[3]**

2a) let $f(x) = \frac{1}{\sqrt{1+x^2}}$

$f(0) = 1$

$f(0.25) = 0.9701$

$f(0.5) = 0.8944$

$f(0.75) = 0.8$

$f(1) = 0.7071$

$$\text{Area} = \frac{0.25}{2}\left(1 + 0.7071 + 2(0.9701 + 0.8944 + 0.8)\right)$$

$$= 0.8795$$

(b) Explain how the trapezium rule might be used to give a better approximation to the integral given in part **(a)**. **[1]**

b) Use smaller intervals

3 **In this question you must show detailed reasoning.**

Given that $5\sin 2x = 3\cos x$, where $0^\circ < x < 90^\circ$, find the exact value of $\sin x$. **[4]**

3 $5\sin 2x = 3\cos x$

$5(2\sin x\cos x) = 3\cos x$

$\cos x(10\sin x - 3) = 0$

$\cos x = 0$ or $10\sin x - 3 = 0$

no values for $0 < x < 90$ satisfy this

$10\sin x = 3$

$\sin x = \frac{3}{10}$

4 For a small angle θ, where θ is in radians, show that $1+\cos\theta-3\cos^2\theta \approx -1+\frac{5}{2}\theta^2$. **[4]**

4 Use $\cos\theta = 1 - \frac{1}{2}\theta^2$

$1 + \cos\theta - 3\cos^2\theta = 1 + (1 - \frac{1}{2}\theta^2) - 3(1 - \frac{1}{2}\theta^2)^2$

$= 1 + 1 - \frac{1}{2}\theta^2 - 3 + 3\theta^2 - \frac{3}{4}\theta^4$

when θ is small you can neglect the higher order terms of θ^4

$= -1 + \frac{5}{2}\theta^2 - \cancel{\frac{3}{4}\theta^4}$

$\therefore\ 1 + \cos\theta - 3\cos^2\theta \approx -1 + \frac{5}{2}\theta^2$

5 **(a)** Find the first three terms in the expansion of $(1+px)^{\frac{1}{3}}$ in ascending powers of x. **[3]**

5 a) $(1+px)^{\frac{1}{3}} = 1 + \frac{1}{3}px + \frac{(\frac{1}{3})(-\frac{2}{3})(px)^2}{2!}$

$= 1 + \frac{1}{3}px - \frac{1}{9}p^2x^2$

(b) The expansion of $(1+qx)(1+px)^{\frac{1}{3}}$ is $1+x-\frac{2}{9}x^2+\ldots$.

Find the possible values of the constants p and q. **[5]**

b) $(1+qx)(1+px)^{\frac{1}{3}} = (1+qx)(1+\frac{1}{3}px - \frac{1}{9}p^2x^2)$

$= 1 + \frac{1}{3}px - \frac{1}{9}p^2x^2 + qx + \frac{1}{3}pqx^2 - \frac{1}{9}p^2qx^3$

$= 1 + x(\frac{1}{3}p + q) + x^2(\frac{1}{3}pq - \frac{1}{9}p^2) - \frac{1}{9}p^2qx^3$

sub 1 into 2

$3p\left(\frac{3-p}{3}\right) - p^2 = -2$

$3p - p^2 - p^2 = -2$

$0 = 2p^2 - 3p - 2$

$0 = (2p+1)(p-2)$

$p = -\frac{1}{2}$ or $p = 2$

If $p = -\frac{1}{2}$, $q = \frac{3+\frac{1}{2}}{3} = \frac{7}{6}$

If $p = 2$, $q = \frac{3-2}{3} = \frac{1}{3}$

So $p = -\frac{1}{2}$ or 2

$q = \frac{7}{6}$ or $\frac{1}{3}$

6 A curve has equation $y = x^2 + kx - 4x^{-1}$ where k is a constant.

Given that the curve has a minimum point when $x = -2$

- find the value of k **[7]**

6 At the minimum point, $\frac{dy}{dx} = 0$

$y = x^2 + kx - 4x^{-1}$

$\frac{dy}{dx} = 2x + k + 4x^{-2}$

At $x = -2$: $0 = 2(-2) + k + 4(-2)^{-2}$

$0 = -4 + k + 1$

$\underline{k = 3}$

$\frac{d^2y}{dx^2} = 2 - 8x^{-3}$

- show that the curve has a point of inflection which is not a stationary point.

At point of inflection, $\frac{d^2y}{dx^2} = 0$

$0 = 2 - 8x^{-3}$

$8x^{-3} = 2$

$4x^{-3} = 1$

$x = 4^{\frac{1}{3}}$

for $x < 4^{\frac{1}{3}}$, $\frac{d^2y}{dx^2} < 0$

for $x > 4^{\frac{1}{3}}$, $\frac{d^2y}{dx^2} > 0$

when $x = 4^{\frac{1}{3}}$, $\frac{dy}{dx} \neq 0$

hence $x = 4^{\frac{1}{3}}$ is a point of inflection but not a stationary point

7 **(a)** Find $\int 5x^3\sqrt{x^2+1}\,dx$. **[5]**

7 a) $\int 5x^3 \sqrt{x^2+1}\ dx$

let $u = x^2 + 1 \Rightarrow \frac{du}{dx} = 2x$

$\int 5x^3 \sqrt{x^2+1}\ dx = \int 5x^3 \sqrt{u} \cdot \frac{1}{2x}\ du$

$= \int 5x^2 \sqrt{u} \cdot \frac{1}{2}\ du$

$= \frac{5}{2} \int (u-1)\sqrt{u}\ du$

$= \frac{5}{2} \int u^{\frac{3}{2}} - u^{\frac{1}{2}}\ du$

$= \frac{5}{2}\left[u^{\frac{5}{2}} \cdot \frac{2}{5} - \frac{2}{3}u^{\frac{3}{2}}\right] + k$

$= (x^2+1)^{\frac{5}{2}} - \frac{5}{3}(x^2+1)^{\frac{3}{2}} + k$

(b) Find $\int \theta \tan^2 \theta \, d\theta$.

You may use the result $\int \tan \theta \, d\theta = \ln|\sec \theta| + c$. **[5]**

b) $\int \theta \tan^2 \theta \quad d\theta$

use integration by parts

let $u = \theta$ $\qquad \frac{dv}{d\theta} = \tan^2 \theta$

$\frac{du}{d\theta} = 1$

$v = \int \tan^2 \theta = \int \sec^2 \theta - 1 \quad d\theta$

$= \tan \theta - \theta$

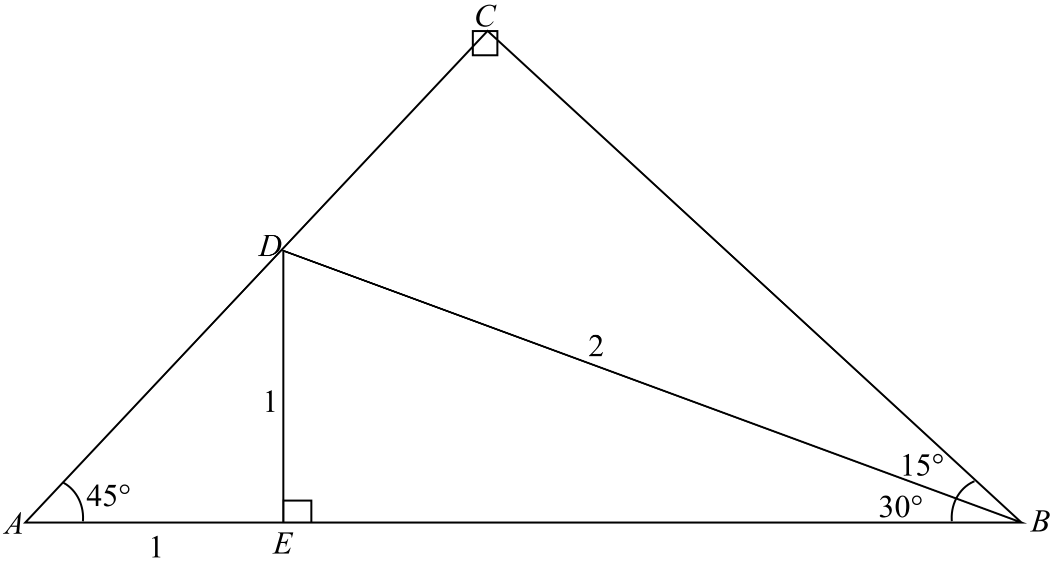
$\int \theta \tan^2 \theta \quad d\theta = \theta(\tan \theta - \theta) - \int \tan \theta - \theta \quad d\theta$

$= \theta \tan \theta - \theta^2 - \ln|\sec \theta| + \frac{1}{2}\theta^2 + c$

$= \theta \tan \theta - \frac{1}{2}\theta^2 - \ln|\sec \theta| + c$

8 **In this question you must show detailed reasoning.**

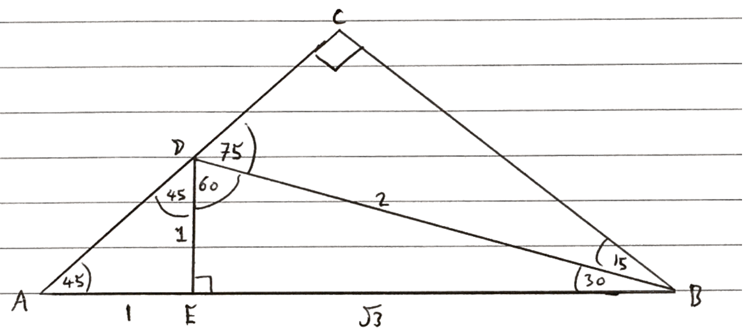
The diagram shows triangle ABC.



The angles CAB and ABC are each 45°, and angle $ACB = 90^\circ$.
The points D and E lie on AC and AB respectively. $AE = DE = 1$, $DB = 2$.
Angle $BED = 90^\circ$, angle $EBD = 30^\circ$ and angle $DBC = 15^\circ$.

(a) Show that $BC = \dfrac{\sqrt{2}+\sqrt{6}}{2}$. **[3]**

8 a)



$BE = \sqrt{2^2 - 1^2} = \sqrt{3}$

$\cos(ABC) = \dfrac{BC}{AB}$

$$\frac{1+\sqrt{3}}{\sqrt{2}} = BC$$

$$BC = \frac{(1+\sqrt{3})}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$BC = \frac{\sqrt{2}+\sqrt{6}}{2}$$

(b) By considering triangle *BCD*, show that $\sin 15^\circ = \frac{\sqrt{6}-\sqrt{2}}{4}$. **[3]**

b) ADC is isosceles so BC = AC

$$BC = AC$$

$$\frac{\sqrt{2}+\sqrt{6}}{2} = AD + DC$$

$$\frac{\sqrt{2}+\sqrt{6}}{2} = \sqrt{2} + DC$$

$$DC = \frac{\sqrt{2}+\sqrt{6}}{2} - \sqrt{2}$$

$$DC = \frac{\sqrt{6}-\sqrt{2}}{2}$$

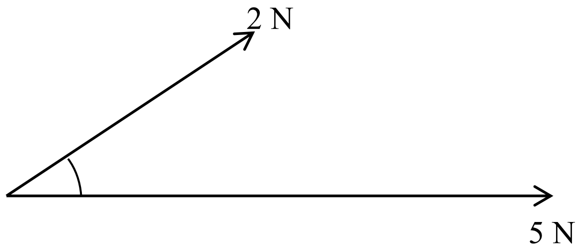
$$\sin 15 = \frac{CD}{BD} = \frac{\sqrt{6}-\sqrt{2}}{2} \div 2$$

$$\sin 15 = \frac{\sqrt{6}-\sqrt{2}}{4}$$

Section B: Mechanics

Answer **all** the questions

9 Two forces, of magnitudes 2 N and 5 N, act on a particle in the directions shown in the diagram below.



(a) Calculate the magnitude of the resultant force on the particle. **[3]**

9 a) $R(\rightarrow)$: $5 + 2\cos 40$

$= 6.5321$

$R(\uparrow)$: $2\sin 40$

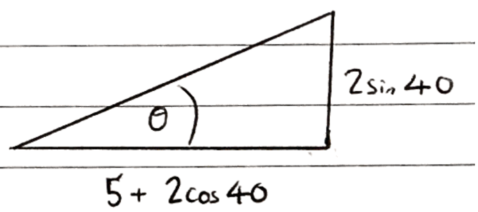
$= 1.2856$

Magnitude $= \sqrt{6.5321^2 + 1.2856^2}$

$= 6.657\text{ N}$

(b) Calculate the angle between this resultant force and the force of magnitude 5 N. **[1]**

b)



$\tan\theta = \dfrac{2\sin 40}{5 + 2\cos 40}$

$\theta = \tan^{-1}(0.1968)$

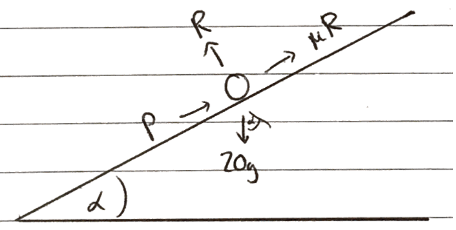
$\theta = 11.13$

10 A body of mass 20 kg is on a rough plane inclined at angle to the horizontal.
The body is held at rest on the plane by the action of a force of magnitude P N.
The force is acting up the plane in a direction parallel to a line of greatest slope of the plane.
The coefficient of friction between the body and the plane is μ.

(a) When $P = 100$, the body is on the point of sliding down the plane.

Show that $g\sin\alpha = g\mu\cos\alpha + 5$. α **[4]**

10



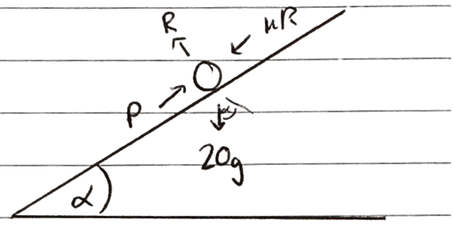
a) when it is on the point of sliding down, the forces parallel to the plane are balanced

$$20g\sin\alpha = \mu R + P$$
$$20g\sin\alpha = \mu \times 20g\cos\alpha + 100$$
$$g\sin\alpha = g\mu\cos\alpha + 5$$

(b) When P is increased to 150, the body is on the point of sliding up the plane.

Use this, and your answer to part **(a)**, to find an expression for in terms of g. **[3]**

b)



Resolve : $20g\sin\alpha + \mu R = P$

$20g\sin\alpha + \mu \times 20g\cos\alpha = 150$

$2g\sin\alpha + 2\mu g\cos\alpha = 15$ ①

From the equation before,

$g\sin\alpha = g\mu\cos\alpha + 5$

$g\mu\cos\alpha = g\sin\alpha - 5$ ②

sub ② into ①

$2g\sin\alpha + 2(g\sin\alpha - 5) = 15$

$2g\sin\alpha + 2g\sin\alpha - 10 = 15$

$4g\sin\alpha = 25$

$\sin\alpha = \frac{25}{4g}$

$\alpha = \sin^{-1}\left(\frac{25}{4g}\right)$

11 In this question the unit vectors **i** and **j** are in the directions east and north respectively.

A particle of mass 0.12 kg is moving so that its position vector **r** metres at time t seconds is given by $\mathbf{r} = 2t^3\mathbf{i} + (5t^2 - 4t)\mathbf{j}$.

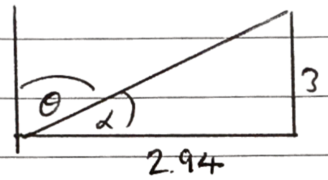
(a) Show that when $t = 0.7$ the bearing on which the particle is moving is approximately $044°$. **[3]**

11 a) $\underline{r} = 2t^3\underline{i} + (5t^2 - 4t)\underline{j}$

$\underline{v} = \frac{d}{dt}\underline{r} = 6t^2\underline{i} + (10t - 4)\underline{j}$

At $t = 0.7$, $\underline{v} = 6(0.7)^2\underline{i} + (10(0.7) - 4)\underline{j}$

$\underline{v} = 2.94\underline{i} + 3\underline{j}$



$\alpha = \tan^{-1}\left(\frac{3}{2.94}\right) = 45.6$

bearing $= \theta = 90 - 45.6 = 44.4°$

bearing $= 044°$

(b) Find the magnitude of the resultant force acting on the particle at the instant when $t = 0.7$. **[4]**

b) $\underline{a} = \frac{d}{dt}\underline{v} = 12t\,\underline{i} + 10\underline{j}$

At $t = 0.7$, $\underline{a} = 12(0.7)\underline{i} + 10\underline{j}$

$\underline{a} = 8.4\underline{i} + 10\underline{j}$

Using $F = ma$:

$F = 0.12(8.4\underline{i} + 10\underline{j})$

$F = 1.008\underline{i} + 1.2\underline{j}$

Magnitude $= \sqrt{1.008^2 + 1.2^2} = 1.57\text{ N}$

So the total height above the ground is

$2.108 + 1.5 = 3.608\text{m}$

ii horizontally : $u\cos 40 = \frac{s}{t}$

$t = \frac{6}{10\cos 40}$

vertically : $s = s$

$u = u\sin 40$

$v = -$

$a = -9.8$

$t = \frac{6}{10\cos 40}$

$s = ut + \frac{1}{2}at^2$

$s = u\sin 40\left(\frac{6}{10\cos 40}\right) + \frac{1}{2}(-9.8)\left(\frac{6}{10\cos 40}\right)^2$

$s = 2.029$

This is $1.5 + 2.029 = 3.529$ m above the ground
This is $3.529 - 2.5 = 1.029$ m above the hoop

$= 1.03$m

b) ~~ver~~ horizontally : $u\cos 40 = \frac{s}{t}$

(c) Determine the times at which the particle is moving on a bearing of 045°. **[2]**

c) This happens when the i component of velocity is equal to the j component of velocity

$6t^2 = 10t - 4$

$6t^2 - 10t + 4 = 0$

$3t^2 - 5t + 2 = 0$

$(3t - 2)(t - 1)$

$t = \frac{2}{3}$ or $t = 1$

both values of t are valid

12 A girl is practising netball.
She throws the ball from a height of 1.5 m above horizontal ground and aims to get the ball through a hoop.
The hoop is 2.5 m vertically above the ground and is 6 m horizontally from the point of projection.

The situation is modelled as follows.

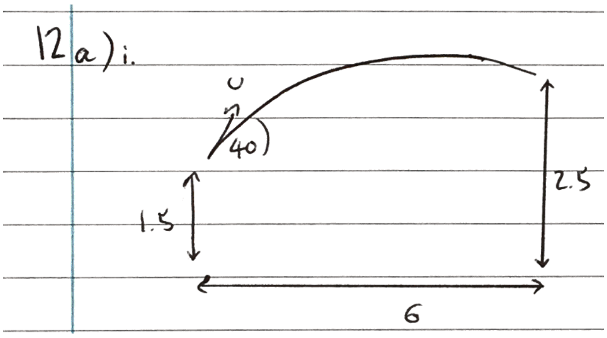
- The initial velocity of the ball has magnitude U m s^{-1}.
- The angle of projection is 40°.
- The ball is modelled as a particle.
- The hoop is modelled as a point.

This is shown on the diagram below.

(a) For $U = 10$, find

(i) the greatest height above the ground reached by the ball **[5]**

12 a) i.



The max height is when the vertical component of velocity is zero
Use suvat on the vertical motion

$s = s$
$u = U\sin 40$
$v = 0$
$a = -9.8$
$t = -$

vertical component of $U = 10\sin 40$
vertical component of velocity $= 10\sin 40 - gt = 0$
$t = 0.656$
vertical displacement $= 10\sin 40t - \frac{1}{2}gt^2$
$2.11 + 1.5 = \underline{3.61\text{m}}$

(ii) the distance between the ball and the hoop when the ball is vertically above the hoop. **[4]**

a)ii) horizontal component of $U = 10\cos 40$

$6 = 10\cos 40t$
$t = 0.783$
$(2.028586218 + 1.5) - 2.5 = 1.03\text{m}$

(b) Calculate the value of U which allows her to hit the hoop. **[3]**

$t = \frac{6}{U\cos 40}$

horizontally : $s = 1$ $(2.5 - 1.5 = 1)$

$u = U\sin 40$

$v = -$

$a = -9.8$

$t = \frac{6}{U\cos 40}$

$s = ut + \frac{1}{2}at^2$

$1 = U\sin 40\left(\frac{6}{U\cos 40}\right) + \frac{1}{2}(-9.8)\left(\frac{6}{U\cos 40}\right)^2$

$1 = \frac{6\sin 40}{\cos 40} - 4.9\left(\frac{36}{U^2\cos^2 40}\right)$

$\frac{176.4}{U^2\cos^2 40} = \frac{6\sin 40}{\cos 40} - 1$

$\frac{300.6}{U^2} = 4.0345$

$U^2 = 74.51$

$U = 8.63$

(c) How appropriate is this model for predicting the path of the ball when it is thrown by the girl? **[1]**

c) Not very appropiate since it does not take air resistance into account which will slow the ball down

(d) Suggest one improvement that might be made to this model. **[1]**

d) Model the ball as an object with air resistance

13 Particle A, of mass m kg, lies on the plane Π_1 inclined at an angle of $\tan^{-1}\frac{3}{4}$ to the horizontal.

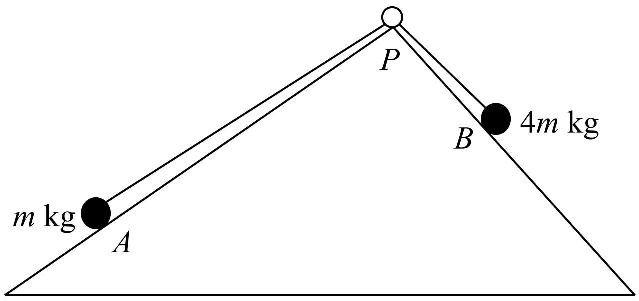
Particle B, of $4m$ kg, lies on the plane Π_2 inclined at an angle of $\tan^{-1}\frac{4}{3}$ to the horizontal.

The particles are attached to the ends of a light inextensible string which passes over a smooth pulley at P.

The coefficient of friction between particle A and Π_1 is and plane Π_2 is smooth.

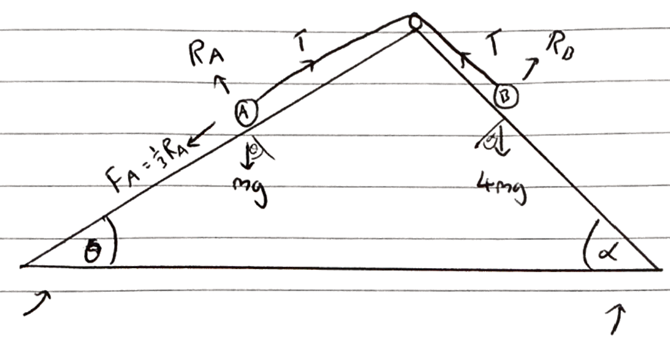
Particle A is initially held at rest such that the string is taut and lies in a line of greatest slope of each plane.

This is shown on the diagram below.

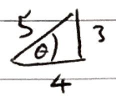


(a) Show that when A is released it accelerates towards the pulley at $\frac{7g}{15}$ m s^{-2}. **[6]**

13 a)

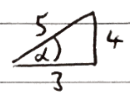


$\theta = \tan^{-1}\left(\frac{3}{4}\right)$ $\quad\quad$ $\alpha = \tan^{-1}\left(\frac{4}{3}\right)$



$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$



$\sin\alpha = \frac{4}{5}$

$\cos\alpha = \frac{3}{5}$

Resolve A perpendicular to the plane : $R_A = mg\cos\theta$

$R_A = \frac{4mg}{5}$

$F_A = \frac{1}{3} R_A = \frac{1}{3} \times \frac{4mg}{5} = \frac{4mg}{15}$

Resolve A parallel to the plane ; use $F = ma$:

$T - \frac{4mg}{15} - mg\sin\theta = ma$

$T - \frac{4mg}{15} - \frac{3mg}{5} = ma$

$T - \frac{13mg}{15} = ma$ ①

Resolve B :

$4mg\sin\alpha - T = 4ma$

$4mg\left(\frac{4}{5}\right) - T = 4ma$

$\frac{16mg}{5} - T = 4ma$ ②

① + ② : $\frac{16mg}{5} - \frac{13mg}{15} = 4ma + ma$

$\frac{48g}{15} - \frac{13g}{15} = 5a$

$\frac{35g}{15} = 5a$

$a = \frac{7g}{15}$

(b) Assuming that A does not reach the pulley, show that it has moved a distance of $\frac{1}{4}$ m when its speed is $\sqrt{\frac{7g}{30}}$ m s^{-1}. **[2]**

b)

$s = s$

$u = 0$

$v = \sqrt{\frac{7g}{30}}$

$a = \frac{7g}{15}$

$t = -$

$$v^2 = u^2 + 2as$$

$$\frac{7g}{30} = 0 + 2\left(\frac{7g}{15}\right)s$$

$$\frac{7g}{30} = \frac{14g}{15}s$$

$$s = 0.25 \text{ m}$$

14 A uniform ladder AB of mass 35 kg and length 7 m rests with its end A on rough horizontal ground and its end B against a rough vertical wall.

The ladder is inclined at an angle of 45° to the horizontal.

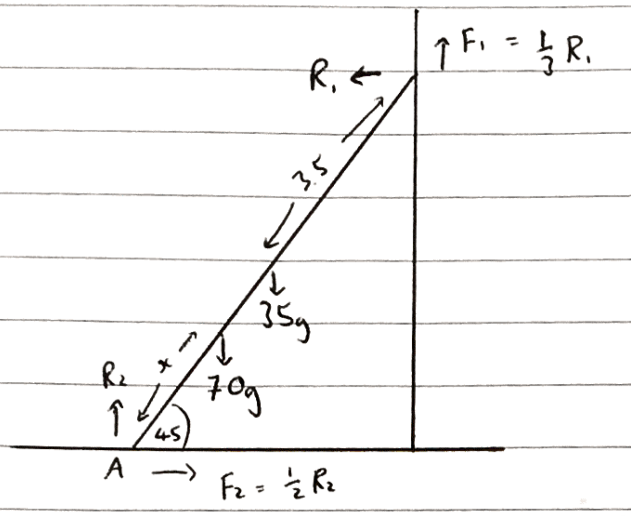
A man of mass 70 kg is standing on the ladder at a point C, which is x metres from A.

The coefficient of friction between the ladder and the wall is and the coefficient of friction between the ladder and the ground is $\frac{1}{2}$.

The system is in limiting equilibrium.

Find x. **[8]**

14



$R(\rightarrow): \quad F_2 = R_1$

$\frac{1}{2}R_2 = R_1$

$R(\uparrow): \quad R_2 + F_1 = 70g + 35g$

$R_2 + \frac{1}{3}R_1 = 105g$

$R_2 + \frac{1}{3}\left(\frac{1}{2}R_2\right) = 105g$

$\frac{7}{6}R_2 = 105g$

$R_2 = 90g$

$R_1 = 45g$

$F_1 = \frac{1}{3} \times 45g$

$= 15g$

$F_2 = \frac{1}{2} \times 90g$

$= 45g$

$\circlearrowright A: \quad x \times 70g\cos 45 + 3.5 \times 35g\cos 45 = 7 \times R_1 \sin 45 + 7 \times F_1 \sin 45$

$70gx\cos 45 + 122.5g\cos 45 = 315g\sin 45 + 105g\sin 45$

$70x\cos 45 = 315\sin 45 + 105\sin 45 - 122.5\cos 45$

$$x = \frac{315\sin 45 + 105\sin 45 - 122.5\cos 45}{70\cos 45}$$

$x = 4.25$

END OF QUESTION PAPER